High-order MHD with Discontinuous Galerkin methods

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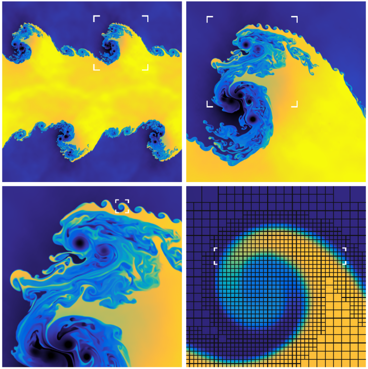
EXAMAG project: simulations of the magnetized Universe

- Numerical schemes for astro simulations
- HPC scalability for large-scale problems

High-order in AREPO AMR

- Greater efficiency, scalability?
- First applications (soon)
    - MHD turbulence
    - $\vec{B}$ amplification

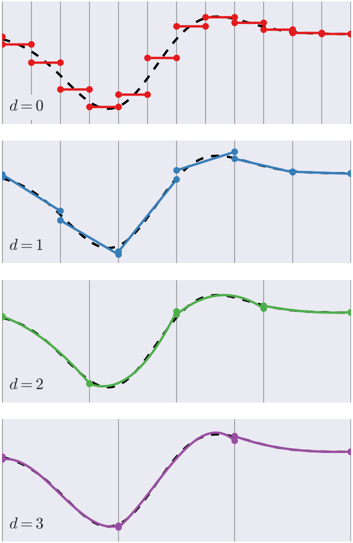


Schaal+2015

This talk:

- DG & shocks/discontinuities
- Illustrative test problems

Discontinuous Galerkin (“DG”) in a Nutshell



- Eulerian AMR mesh, cubic cells
- *High-order* solution description
  - polynomials of degree $\leq d$
- *Discontinuous Galerkin*
  - discontinuous solution at faces
  - evolve polynomial coeffs

Key properties

- Error $= \mathcal{O}\left((\Delta x)^{d+1}\right)$ for smooth solutions
- Discontinuous $\rightarrow$ shock capturing
- Derivatives are local to cell
- AMR $\rightarrow$ spatial dynamic range

DG Scheme: Basic Outline

$$\boxed{\partial_t \boldsymbol{U} + \partial_i \boldsymbol{F}_i(\boldsymbol{U}) = 0; \qquad \boldsymbol{U} = (\rho, \rho\vec{v}, E, \vec{B})}$$

Polynomial expansion in each cell

- $\boldsymbol{U}(x,t) = w_\beta(t)\phi_\beta(x)$, polynomial ϕ_β, $\deg(\phi_\beta) \le d$
  - Legendre polynomials for ρ, $\rho\vec{v}$, E
  - Divergence-free vector polynomials for $\vec{B}$

Weak formulation

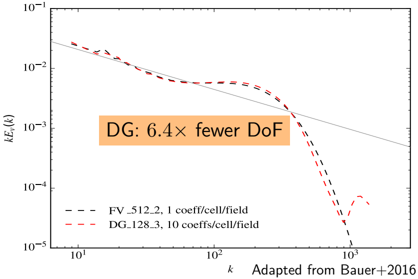
L^2 projection onto ϕ_β + integration by parts:

$$\underbrace{\partial_t \int_K \boldsymbol{U}\cdot\phi_\beta}_{\text{Weights evol}} + \underbrace{\int_{\partial K} \boldsymbol{F}_i \cdot \phi_\beta n_i}_{\text{Face fluxes}} - \underbrace{\int_K \boldsymbol{F}_i \cdot \partial_i \phi_\beta}_{\text{Internal fluxes}} = 0; \qquad \underbrace{\int \to \sum}_{\text{Gaussian quadrature}}$$

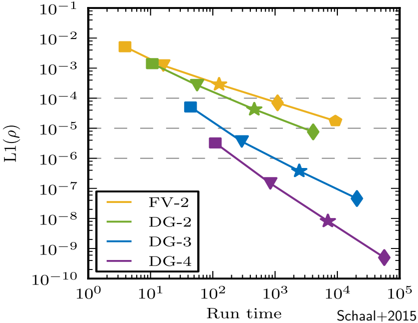
Explicit Runge-Kutta time integration w/ global timesteps

DG: Numerically and Computationally Efficient*

*For smooth enough problems. Talk to your doctor to know if high-order is good for you.



Adapted from Bauer+2016



Schaal+2015

- Very efficient for smooth flows (e.g. subsonic turbulence)

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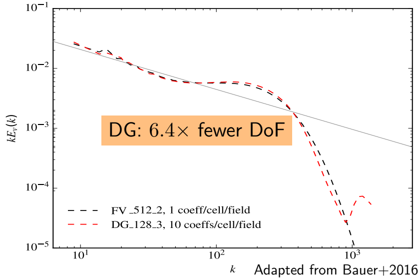

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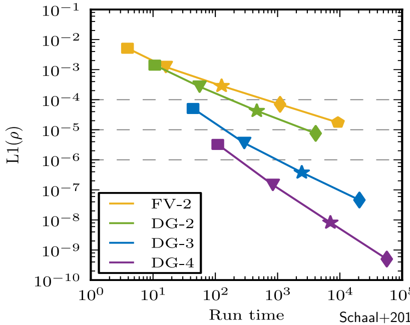
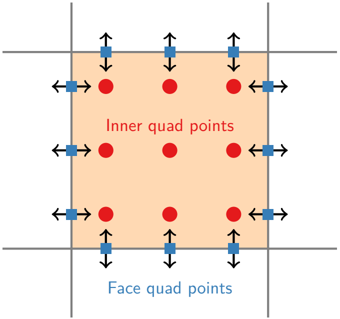
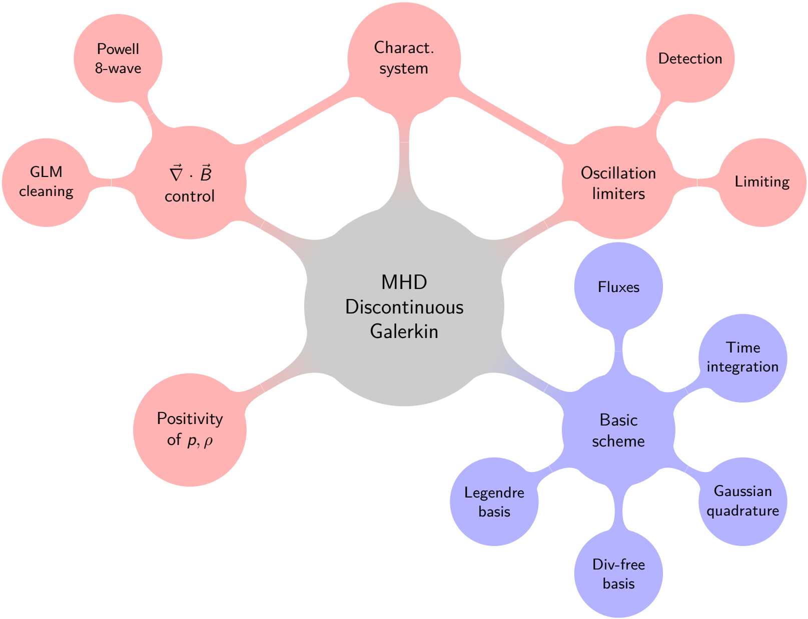


Schaal+2015

- Very efficient for smooth flows (e.g. subsonic turbulence)
- Compute-intensive & local $\rightarrow$ scalability, efficient FP ops



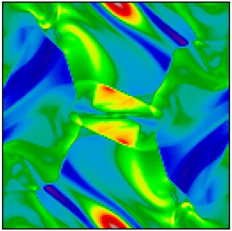
Full DG Scheme: Many Moving Parts!



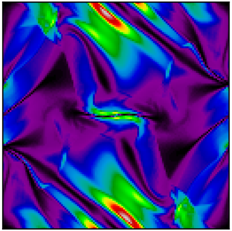
Shock Capturing: Orszag-Tang Vortex

3rd order scheme ($d = 2$)

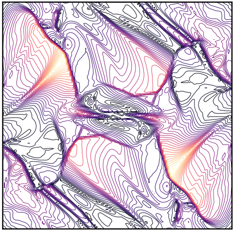
128^2



Density

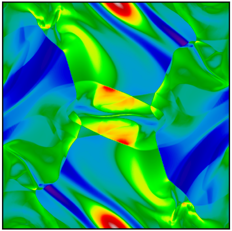


Magnetic pressure



Mach number

512^2



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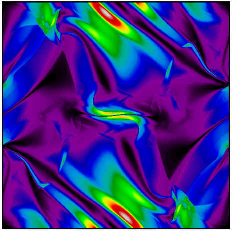

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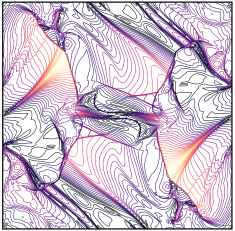
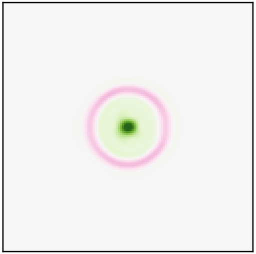


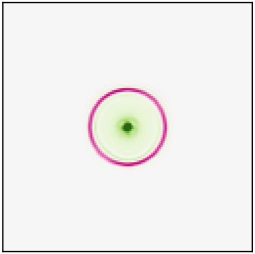
Mach number

DG Can Resolve Sharp Features: $\vec{B}$ Field Loop

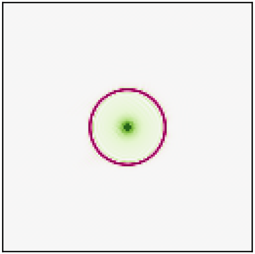
- Advected high-β $\vec{B}$ field loop
- Singular MHD current density $\vec{j}$



DG-2, $t = 2.0$



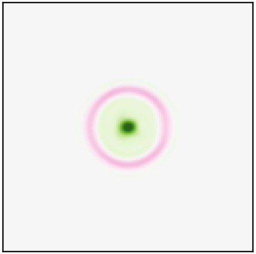
DG-3, $t = 2.0$



DG-4, $t = 2.0$

DG Can Resolve Sharp Features: $\vec{B}$ Field Loop

- Advected high-β $\vec{B}$ field loop
- Singular MHD current density $\vec{j}$
- Shape of loop very well preserved
- Very little noise / ringing



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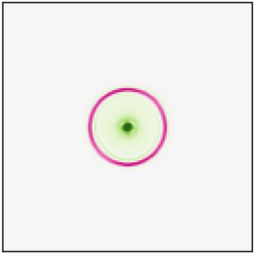

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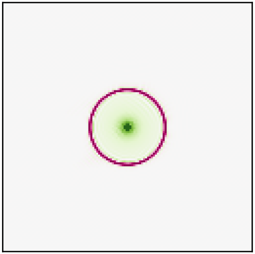

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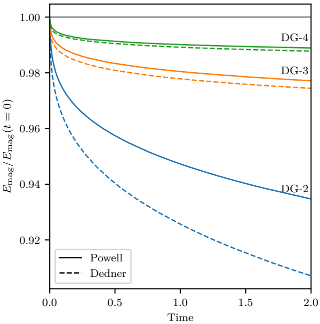
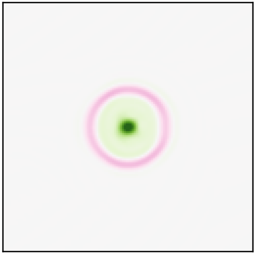
- High-order $\rightarrow$ lower dissipation





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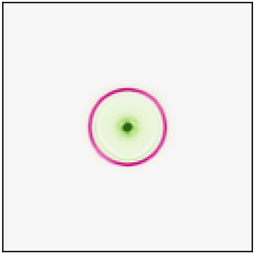

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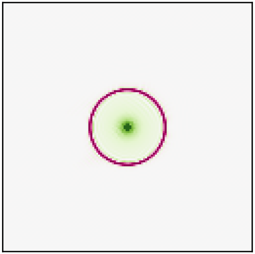

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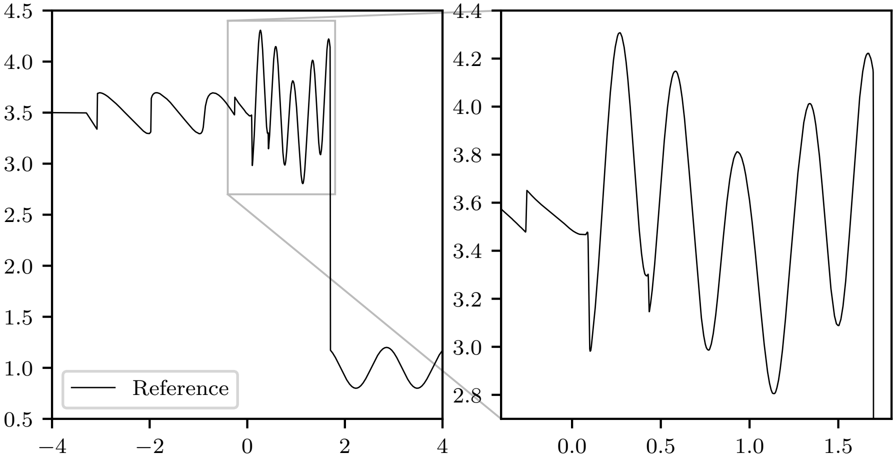
Combined Smooth Flow & Shocks

MHD Shu-Osher shock tube, 128–256 cells



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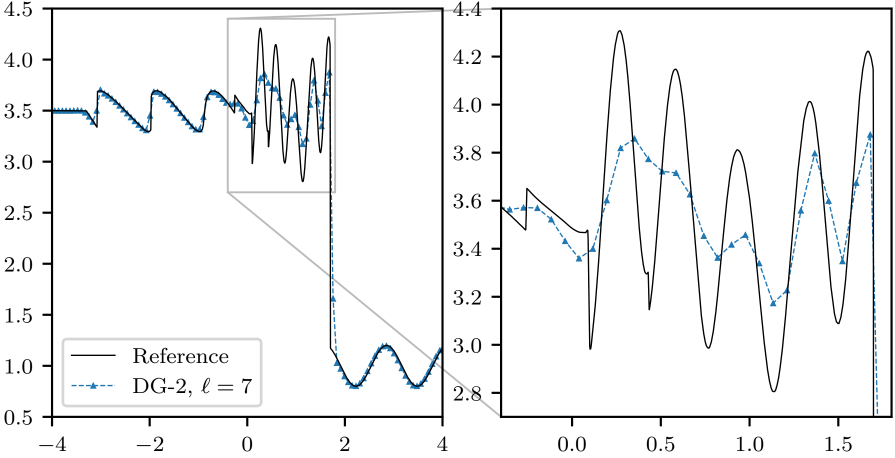

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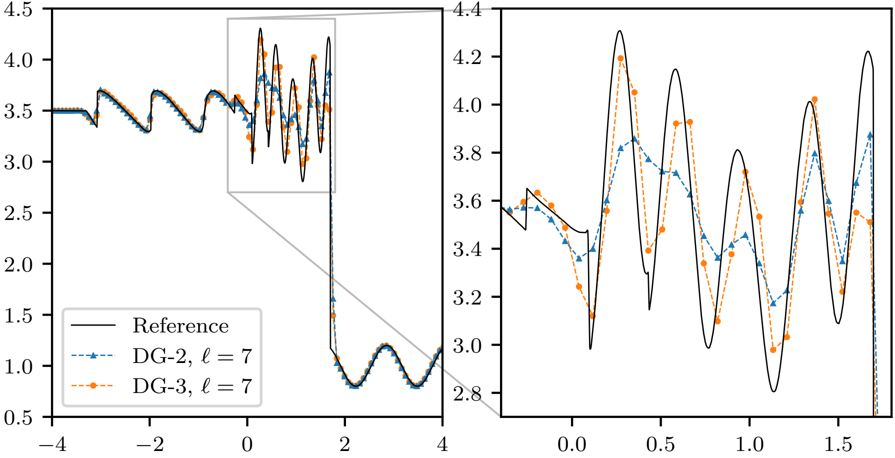

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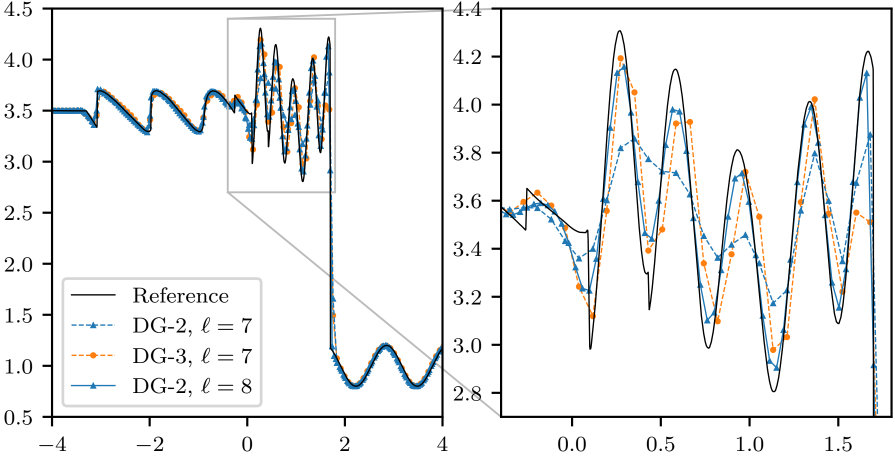

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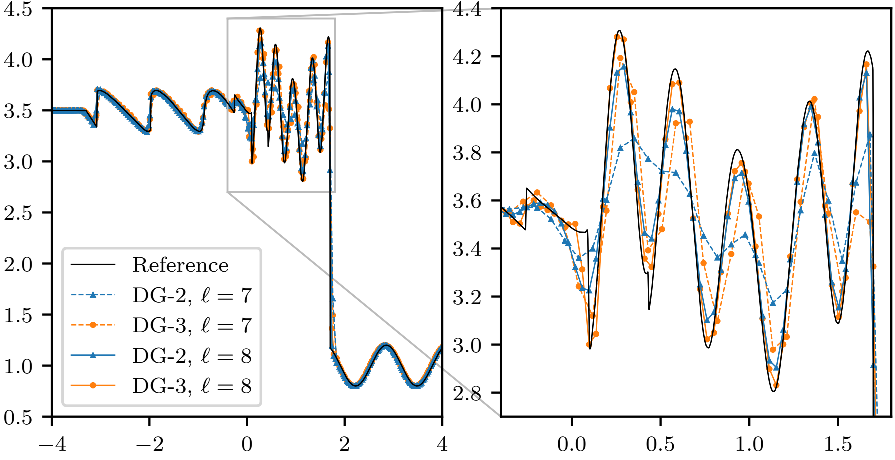
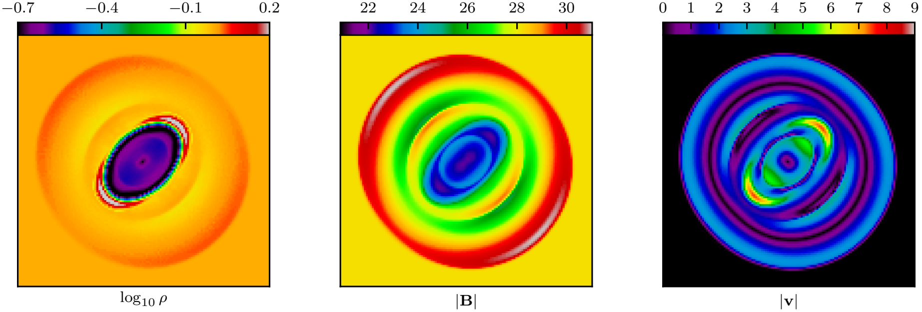


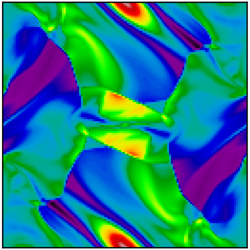
- High-order can achieve greater effective resolution
- Survives in presence of shocks

Handling Very Strong Shocks

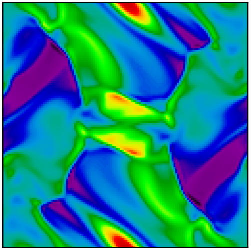
- Magnetized Sedov-like blast in 3D, 3rd order scheme, 128^3
- Plasma-$\beta \sim 2 \times 10^{-4}$
- Pressure ratio $10^4 \rightarrow$ very strong shock
- Positivity limiter is crucial



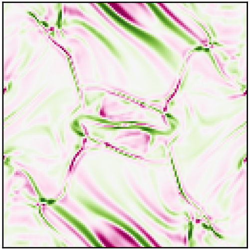
Reduced Advection Errors with MHD Shocks



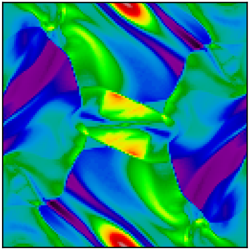
DG-2 stationary



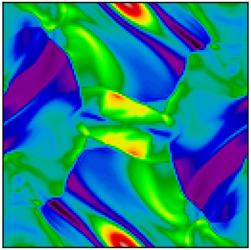
DG-2 advected



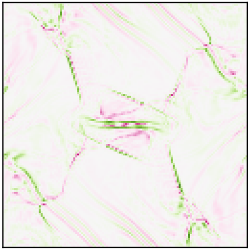
Difference



DG-3 stationary



DG-3 advected



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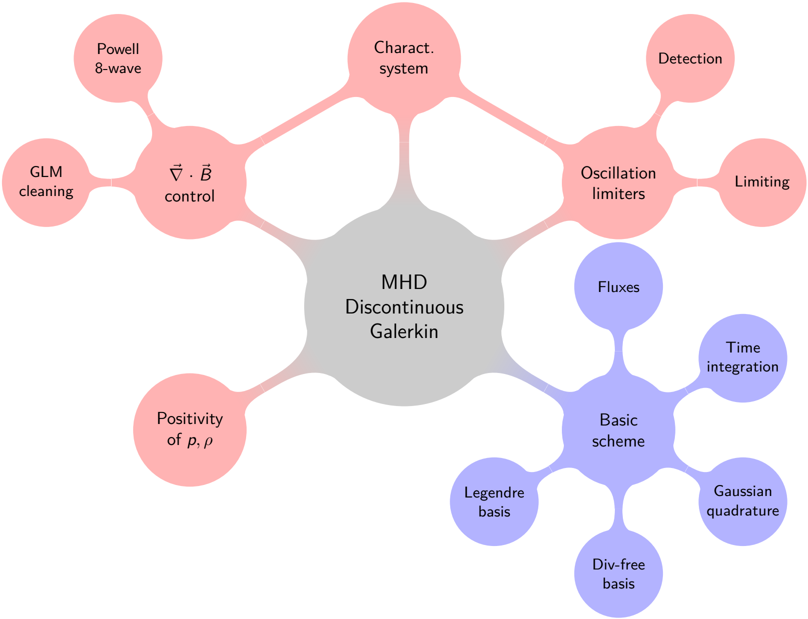

DG for Production Astrophysics Simulations?

Great promises / Increased complexity / How robust & flexible?

High-order methods

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Astro/cosmo codes

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Conclusions

DG fulfills some promises...

- In AREPO: a first working DG-MHD scheme
- Large time-to-solution gains for smooth test problems
- Handles shocks, discontinuous solutions

...but method requires careful caretaking

- Oscillation detection & control (limiters)
- Divergence treatment

Beyond test problems: robustness & flexibility?

- Oscillation limiting for very general problems?
- Extreme dynamic range?
- Crazy, stiff source terms?