# Real Friends Say Hard Things: Numerical Convergence

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# Simulator's Bane: Numerical Convergence

- One can have the best subgrid model for star formation and feedback, but if the simulation results are not numerically converged, one is studying truncation errors…

$$\frac{df}{dx} = \frac{\Delta f}{\Delta x}[\Delta x] + \mathrm{T.E.}$$

- If T.E. is small, then

$$\frac{\Delta f}{\Delta x}[\Delta x_1] \approx \frac{df}{dx} \approx \frac{\Delta f}{\Delta x}[\Delta x_2]$$

# Simulator's Bane: Numerical Convergence

- Individual simulations have fixed spatial $\Delta r$ and mass $M_1$ resolution.
- Any quantity measured in a simulation depends on resolution:

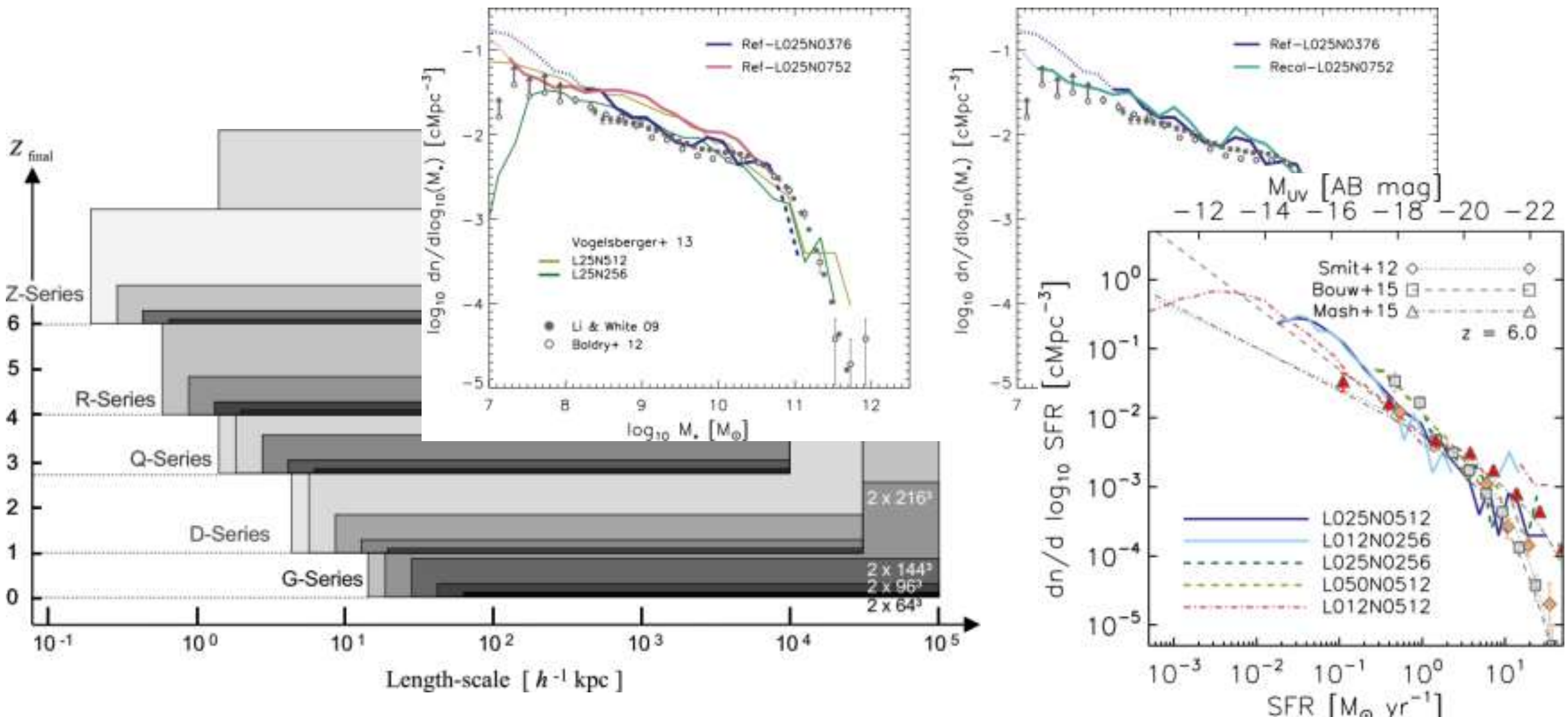
$$Q = Q(\Delta r, M_1)$$

- But only a converged value has physical meaning.

$$\hat{Q} = \lim_{\Delta r \to 0} \lim_{M_1 \to 0} Q(\Delta r, M_1)$$

# The Hard Truth:

- We all want numerical convergence.
- Exploring convergence is expensive, time consuming, and, frankly, plain boring.



# The CROC Project: Getting Ready for JWST++

Cosmic
Reionization
On
Computers

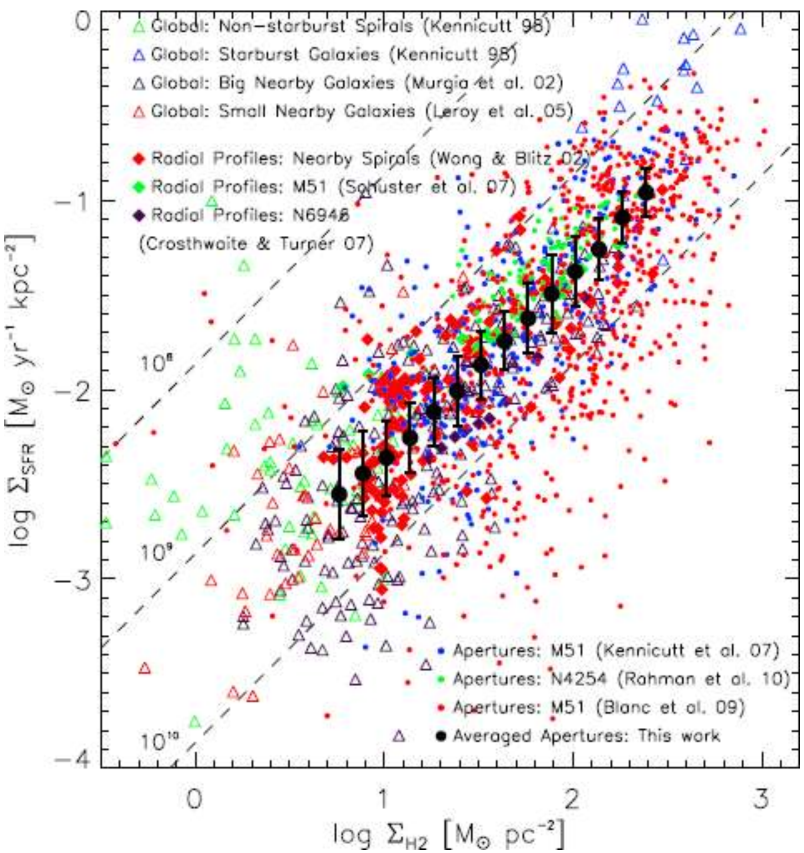
- The goal is to have simulations that realistic in a quantitative sense, and can be used for interpreting the observational data.

| | Yesterday | | Today |
|---|---|---|---|
| | Small box | Large box | |
| physics | full | incomplete | full |
| spatial resolution | high | low | high |
| mass resolution | high | high | high |
| dynamic range | low | high | high |
| volume | small | large | large |

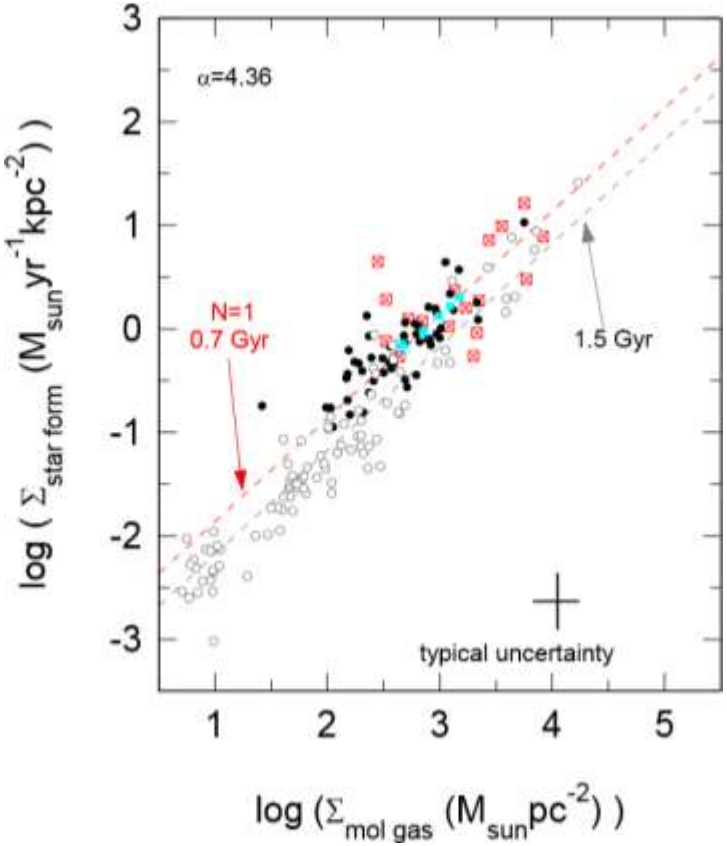
# The CROC Project: Star Formation

Cosmic Reionization On Computers

Star formation correlates well with molecular gas…




z=0 (Bigiel et al 2011)

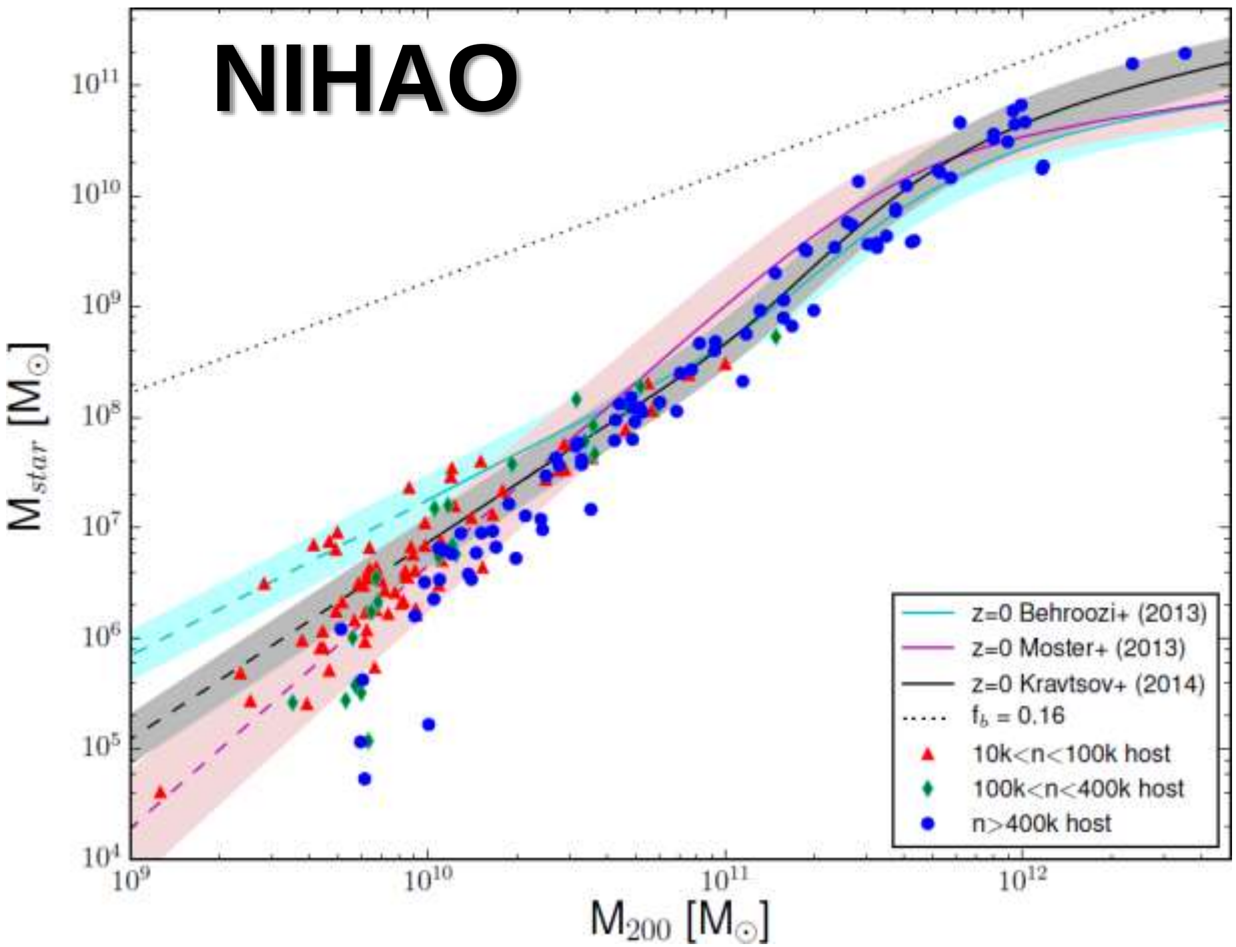


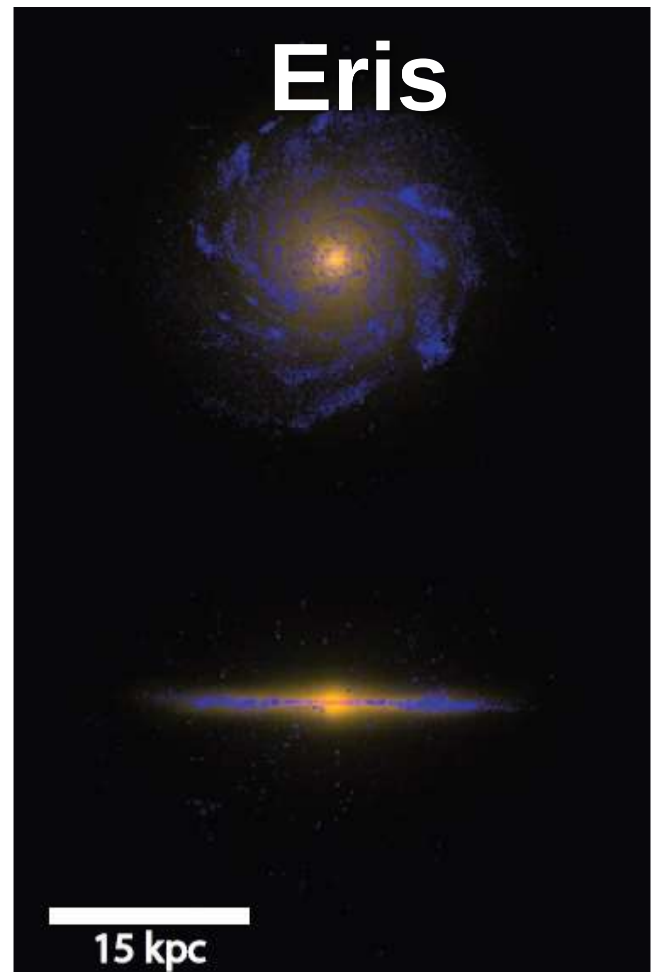

High z (Tacconi et al 2013)

# The CROC Project: Stellar Feedback

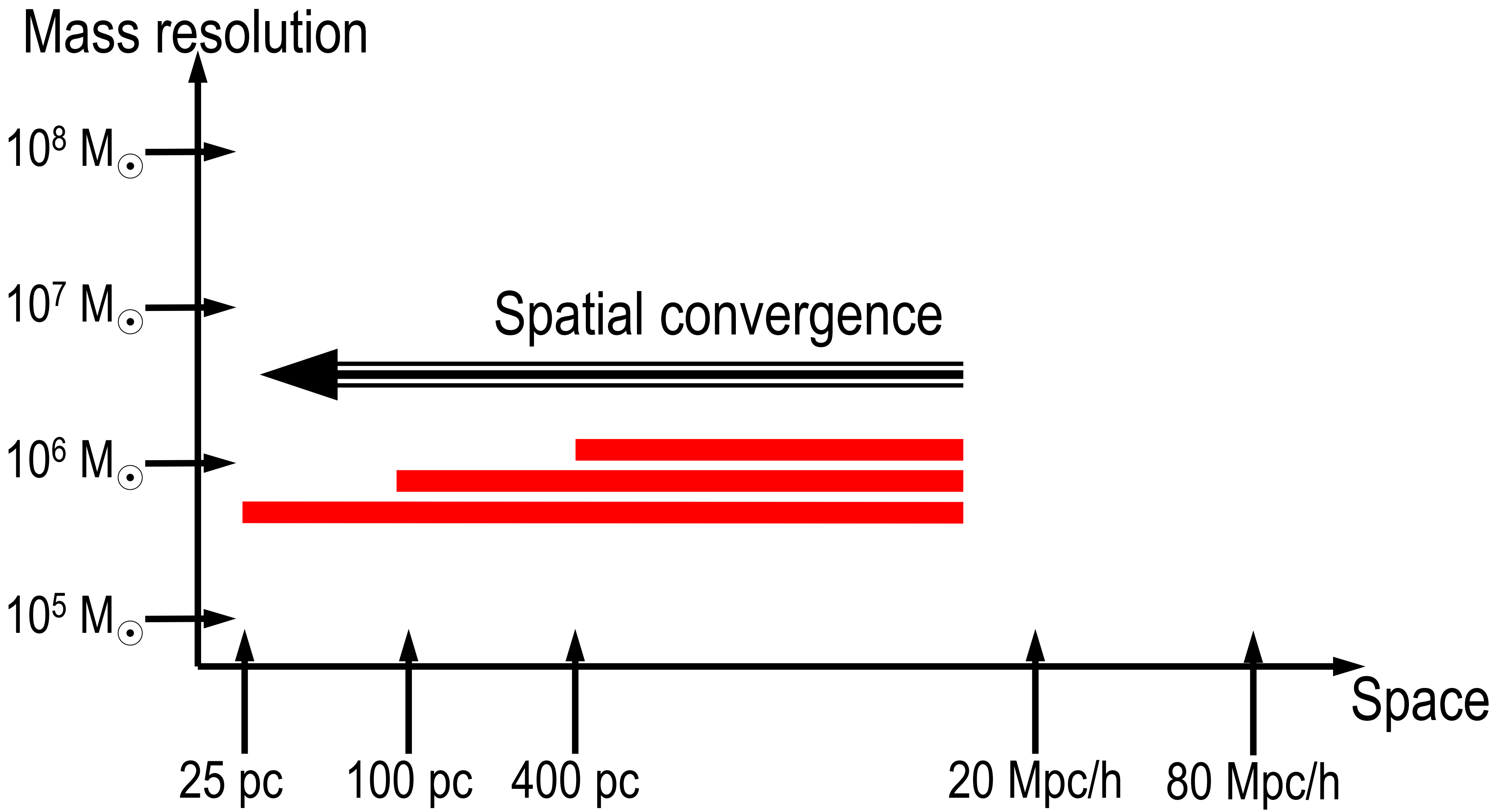
Cosmic Reionization On Computers

"Delayed cooling" is now "industry standard".



# The CROC Project: Convergence Study



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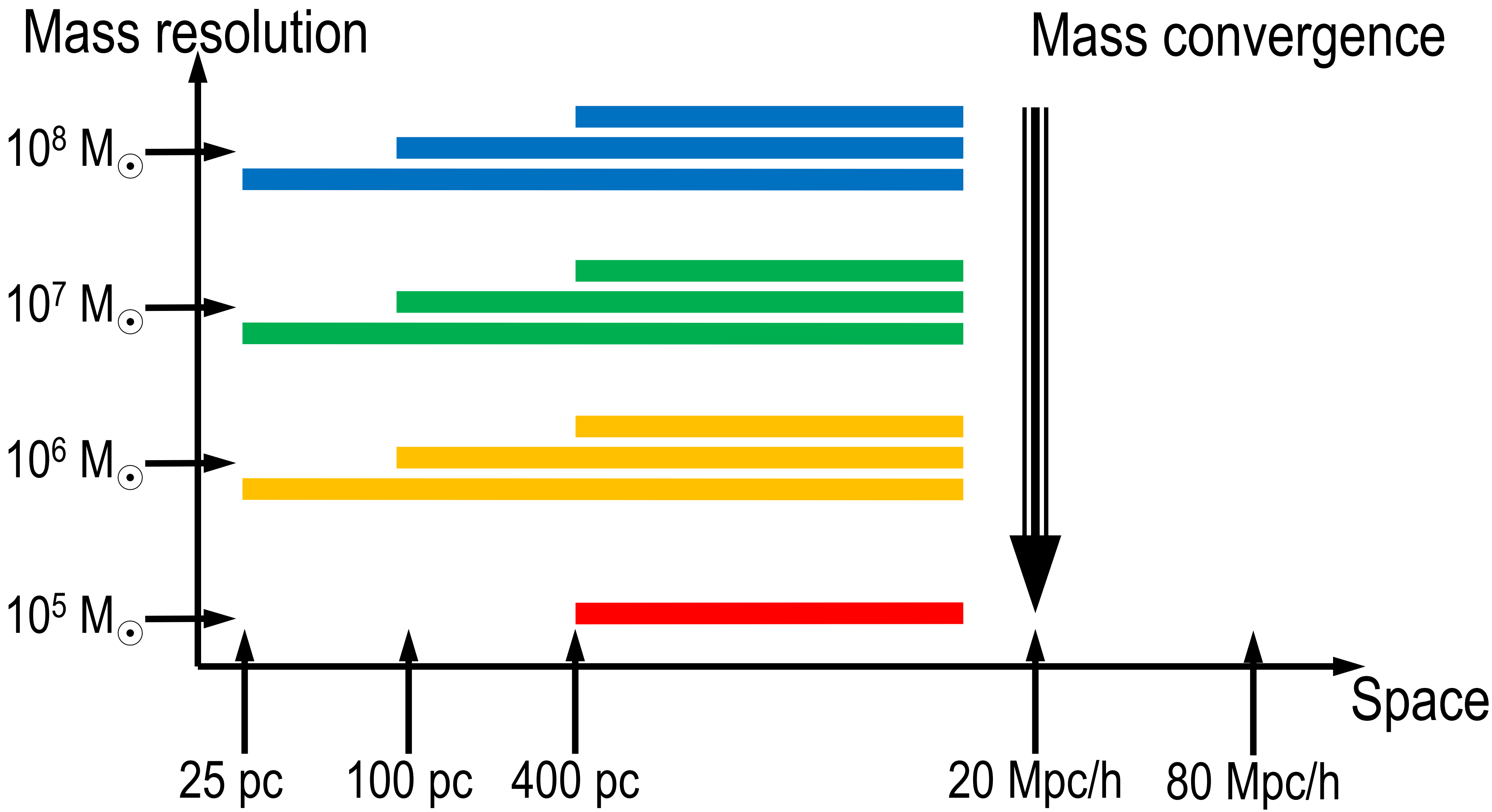
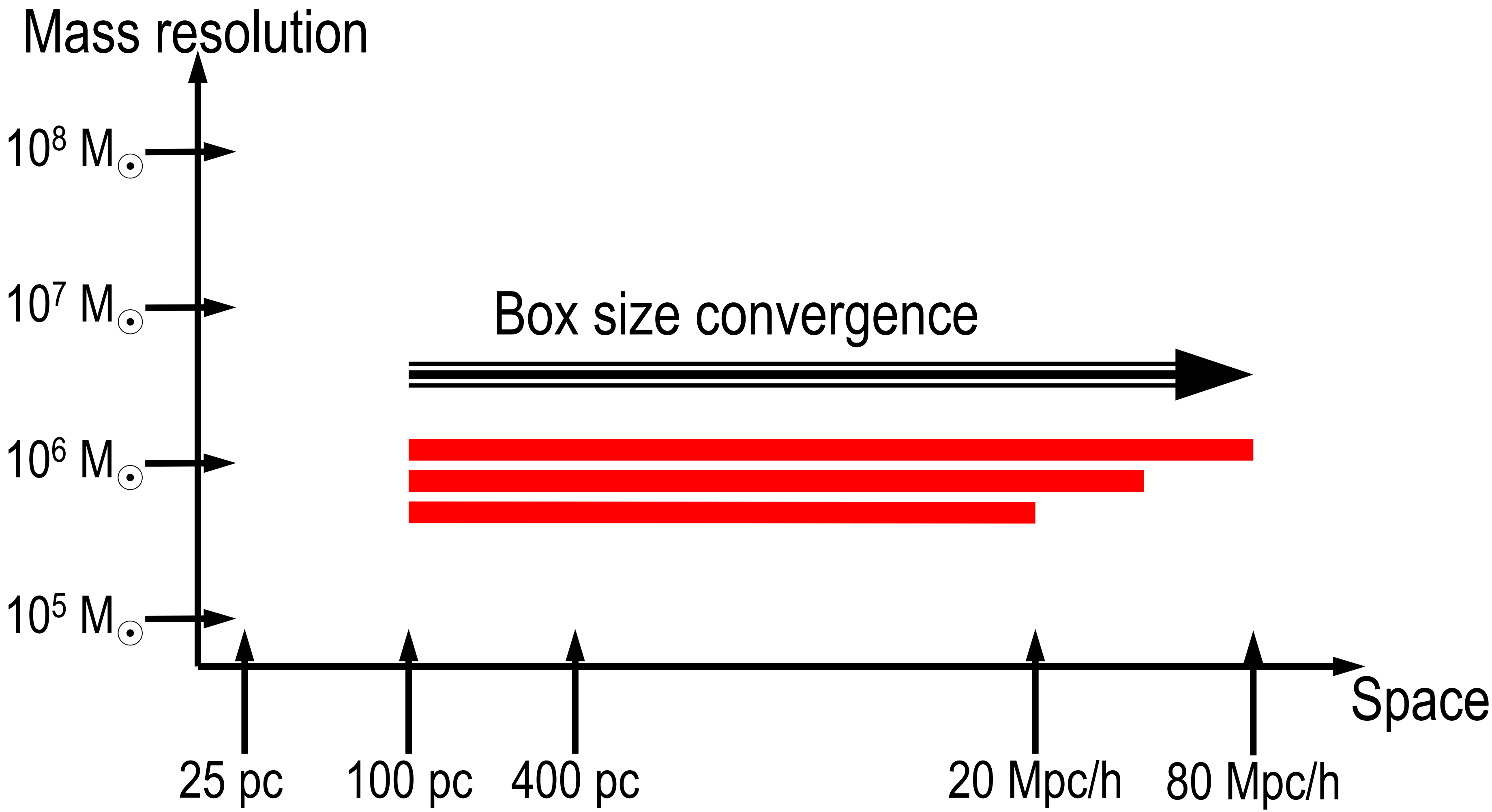
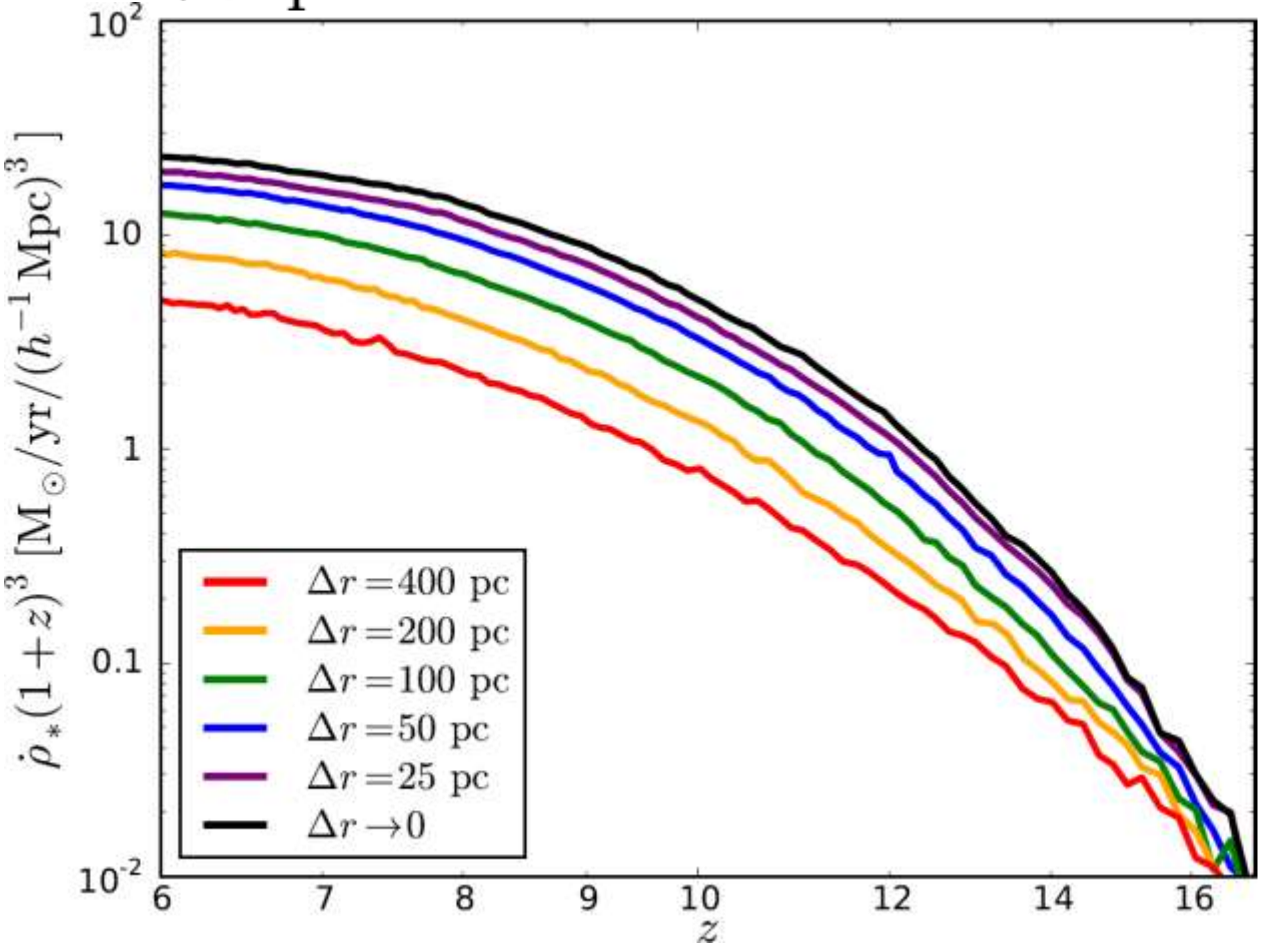


# The CROC Project: Convergence Study



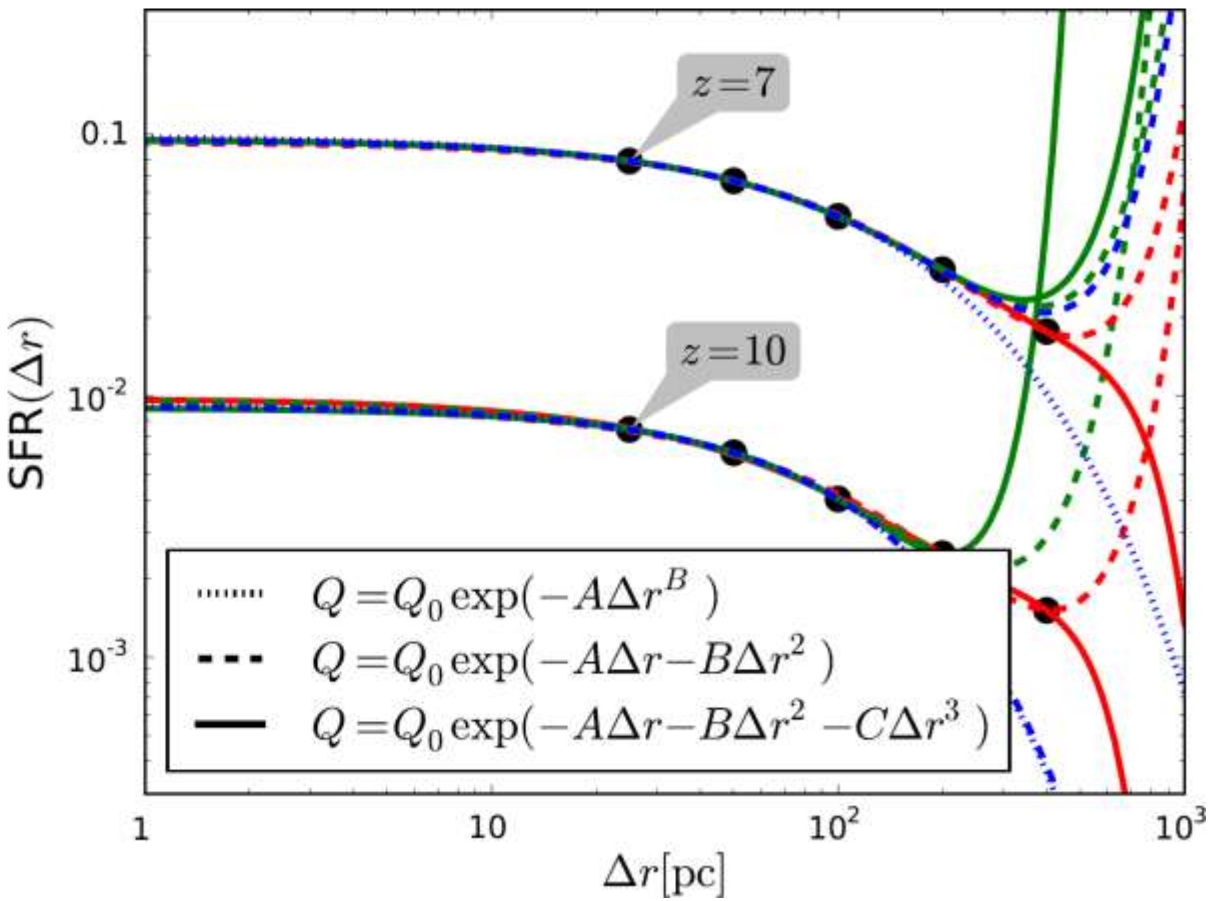
# Global SFR

- Spatial convergence is slow – only reached at $\Delta r < 50\mathrm{pc}$.



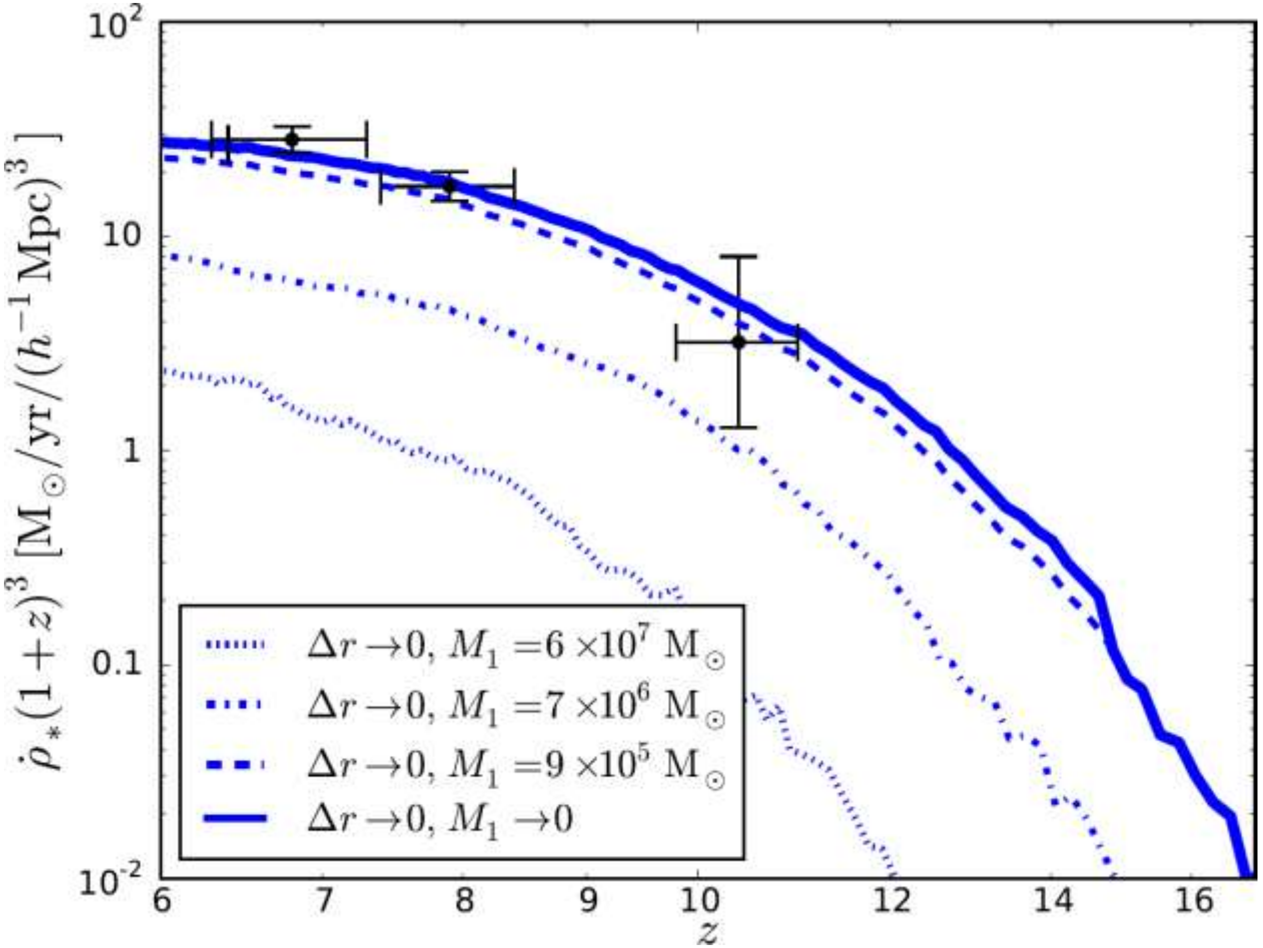
# Global SFR

- When convergence is close, extrapolation to $\Delta r \to 0$ is robust.



# Global SFR

- Extrapolate first in space, then in mass…



# Global SFR

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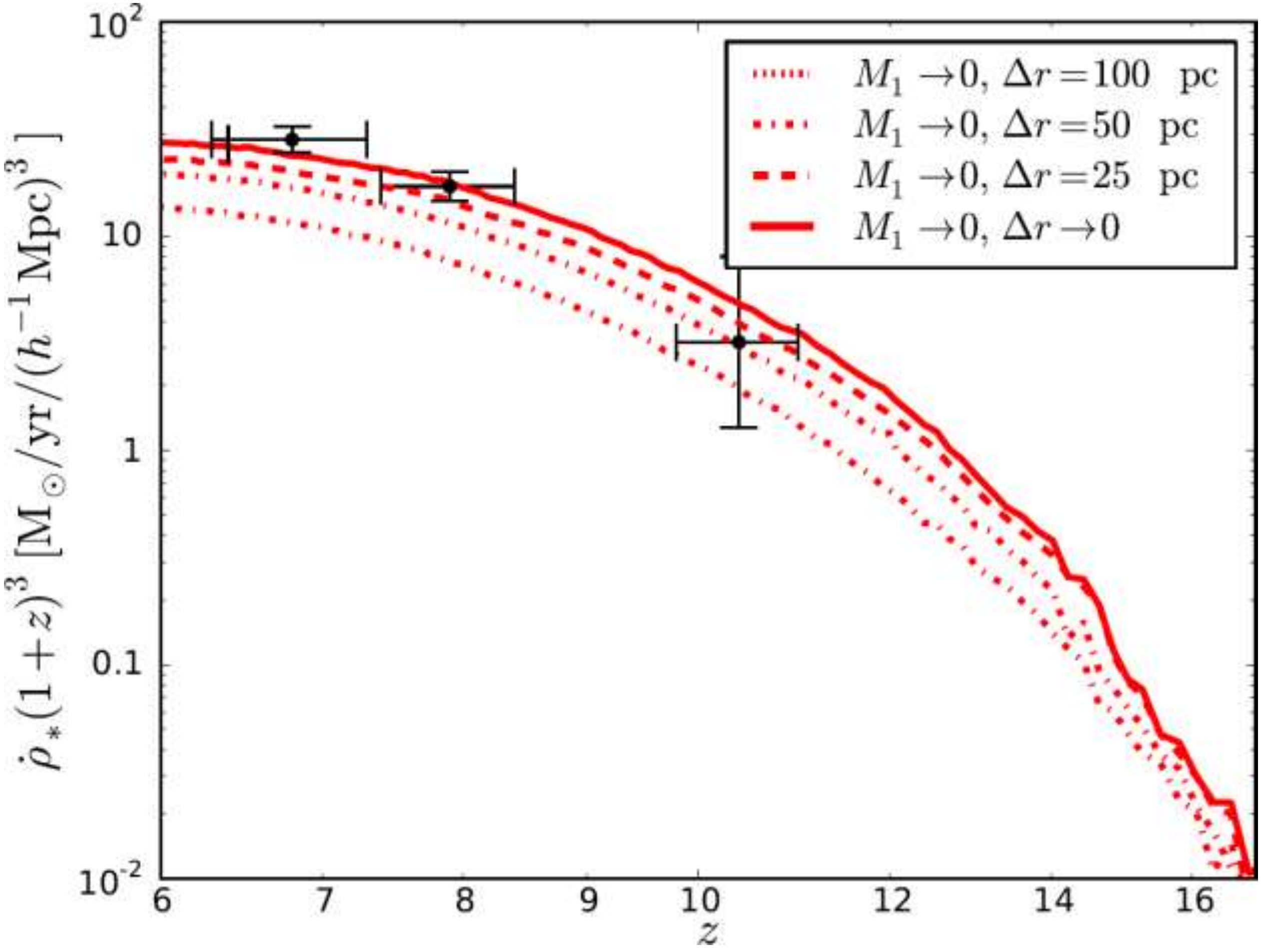
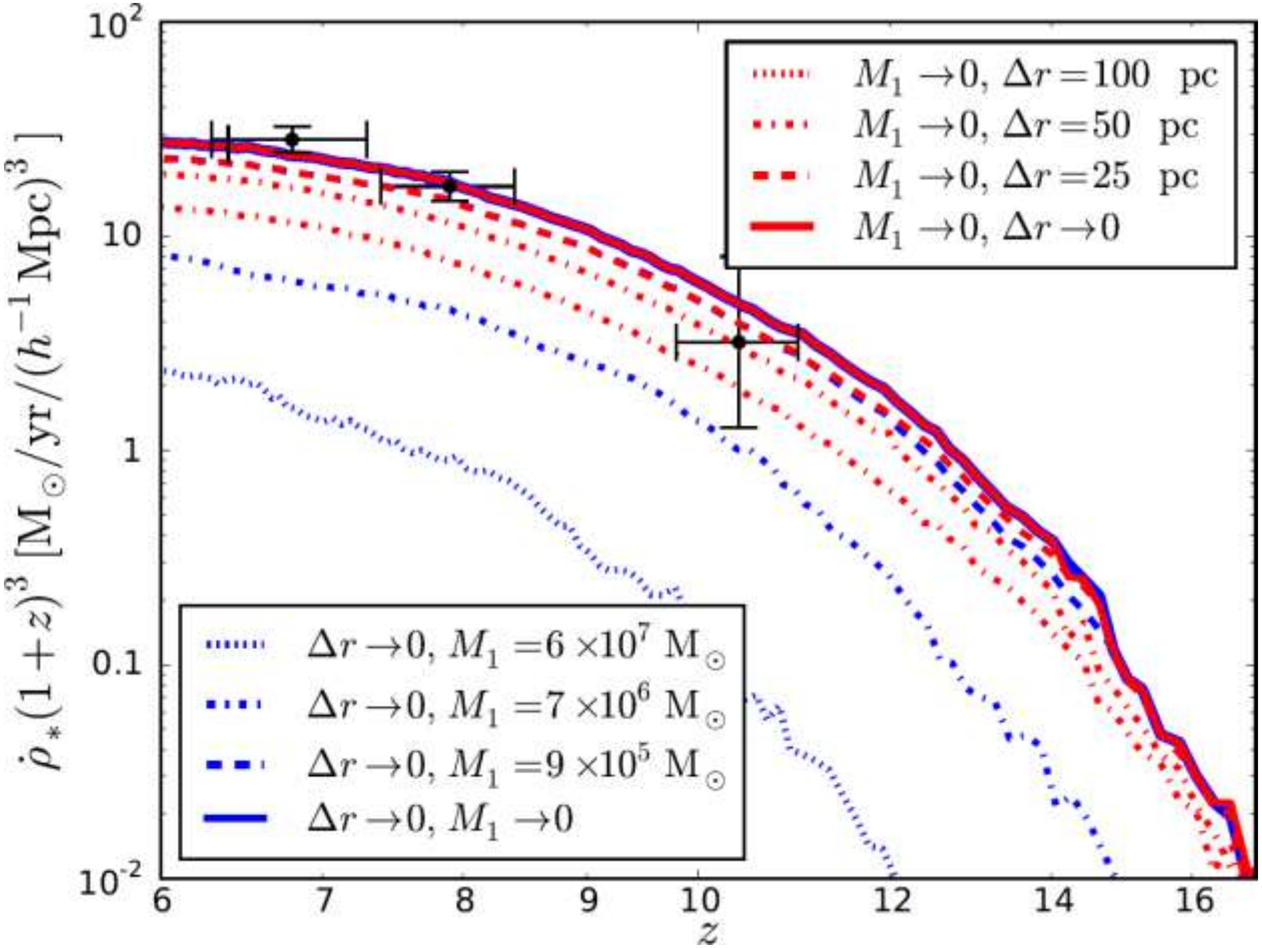


# Global SFR

Cosmic Reionization On Computers

- …and both together.



# Weak Convergence

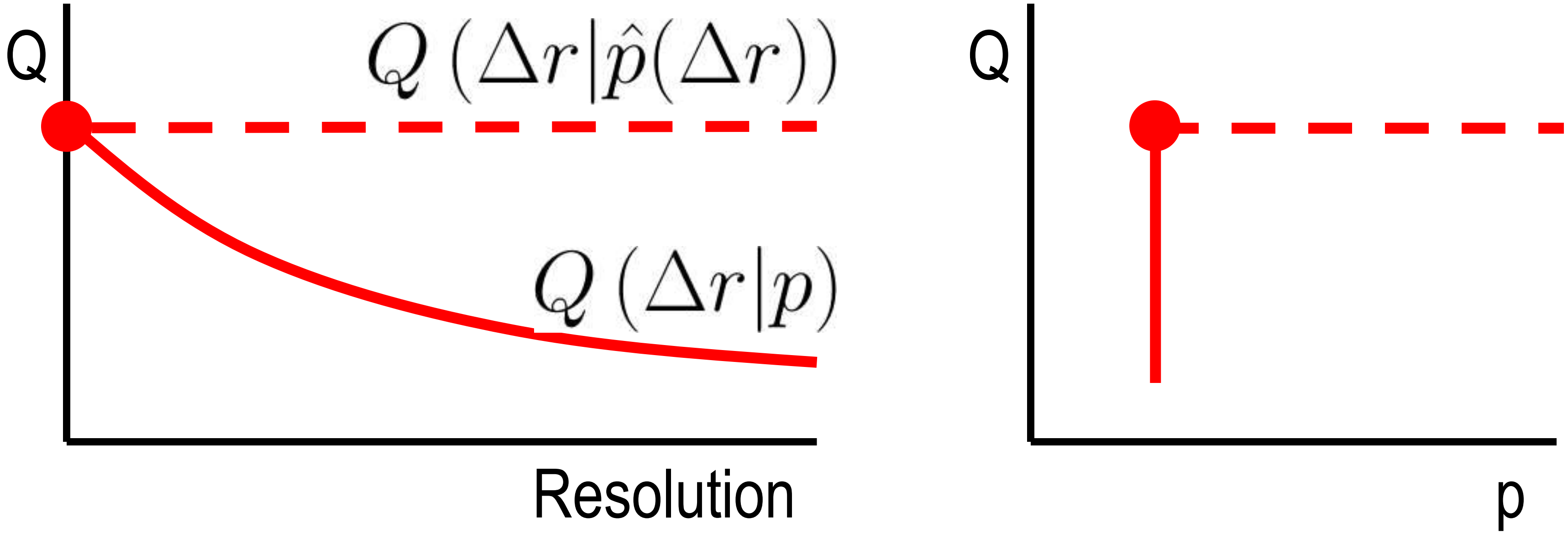
- At production scale, we cannot run the whole convergence ladder – have to reply on “weak convergence”.

$$\hat{Q} = \lim_{\Delta r, M_1 \to 0} Q(\Delta r, M_1 | p_j) \approx Q(\Delta r, M_1 | \hat{p}_j)$$

- As we change the resolution, we adjust the parameters of the model to keep the solution fixed.
- WARNING: if quantity $Q$ is weakly converged, it does not guarantee that some other quantity $H$ is weakly converged too!

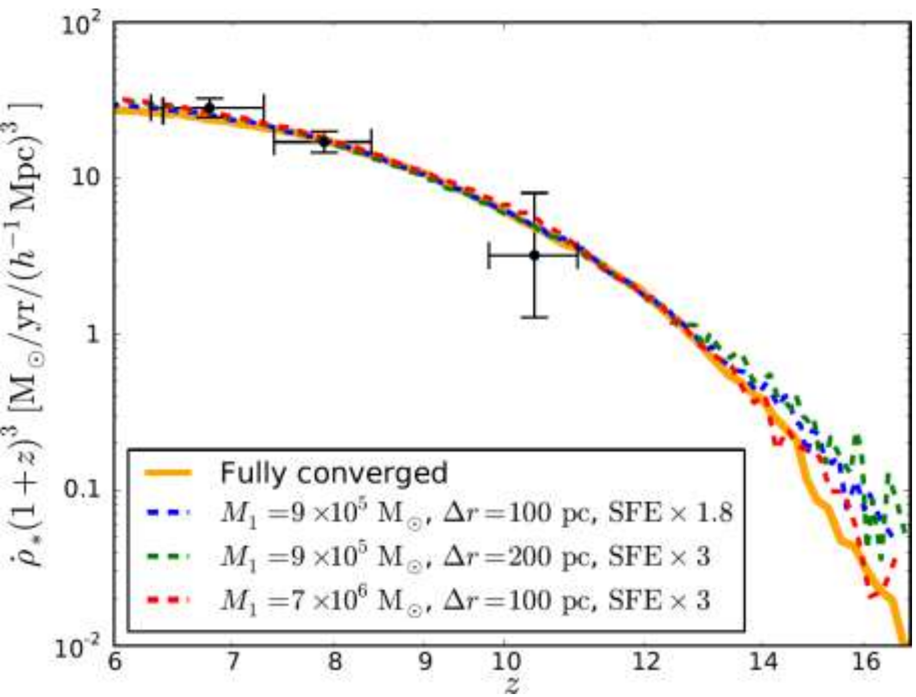
# Weak Convergence

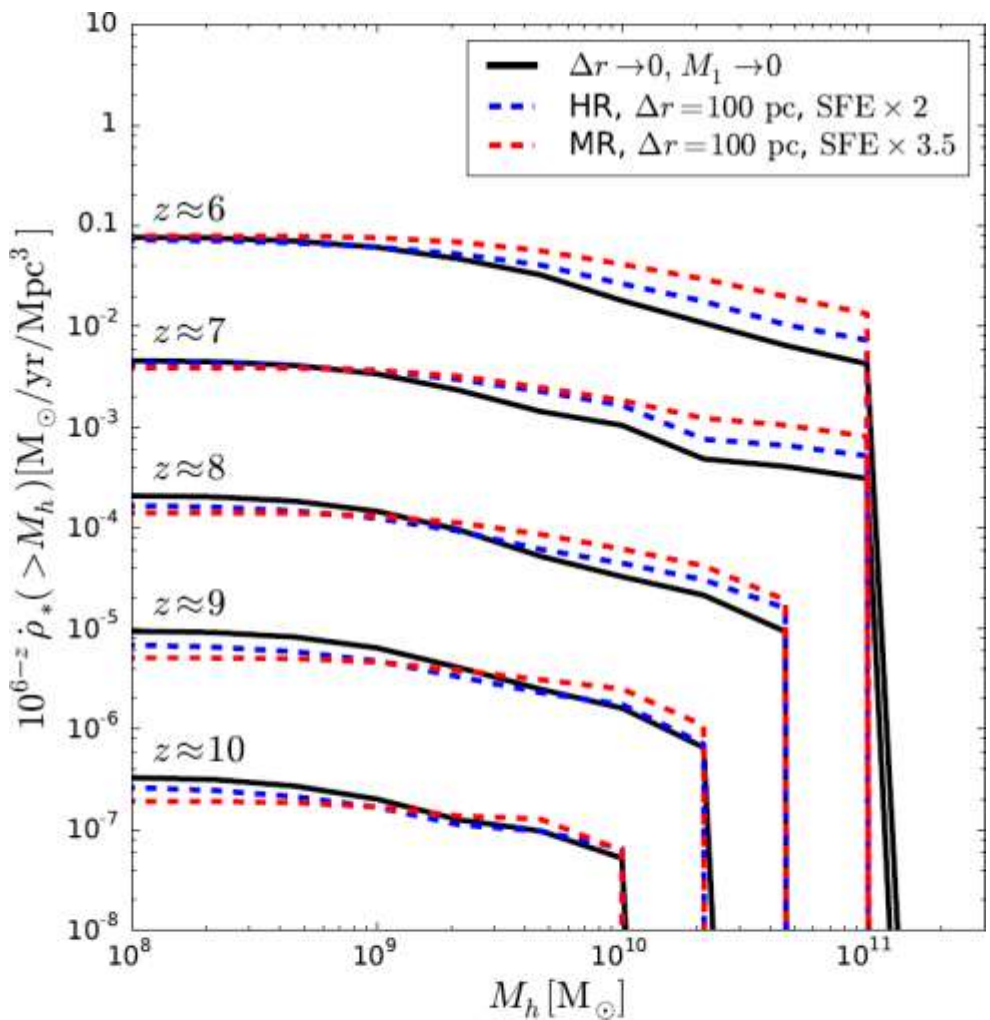
- A graphical view: keeping straight on your path to the goal.



# Weak Convergence

- To weakly converge on global SFR is easy, but it is not enough!



# Weak Convergence

- One needs to rescale SFR in halos of different masses.

$$\dot{\rho}_* \rightarrow \dot{\rho}_* \times W_C$$

- Ideally, one would have

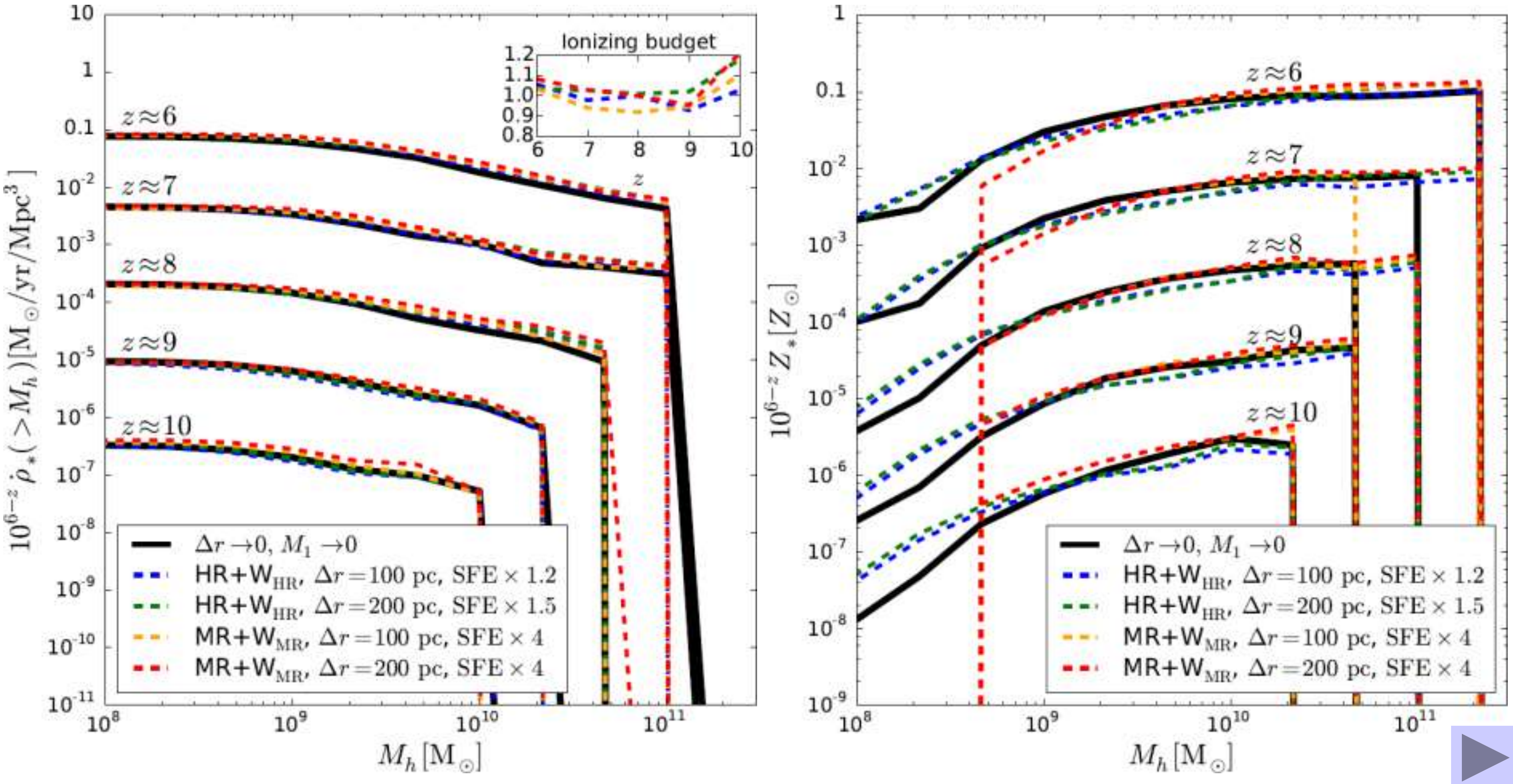
$$W_C = W_C(M_h)$$

- To avoid running a halo finder on-the-fly, make a poor man's proxy:

$$W_C = W_C(\rho)$$

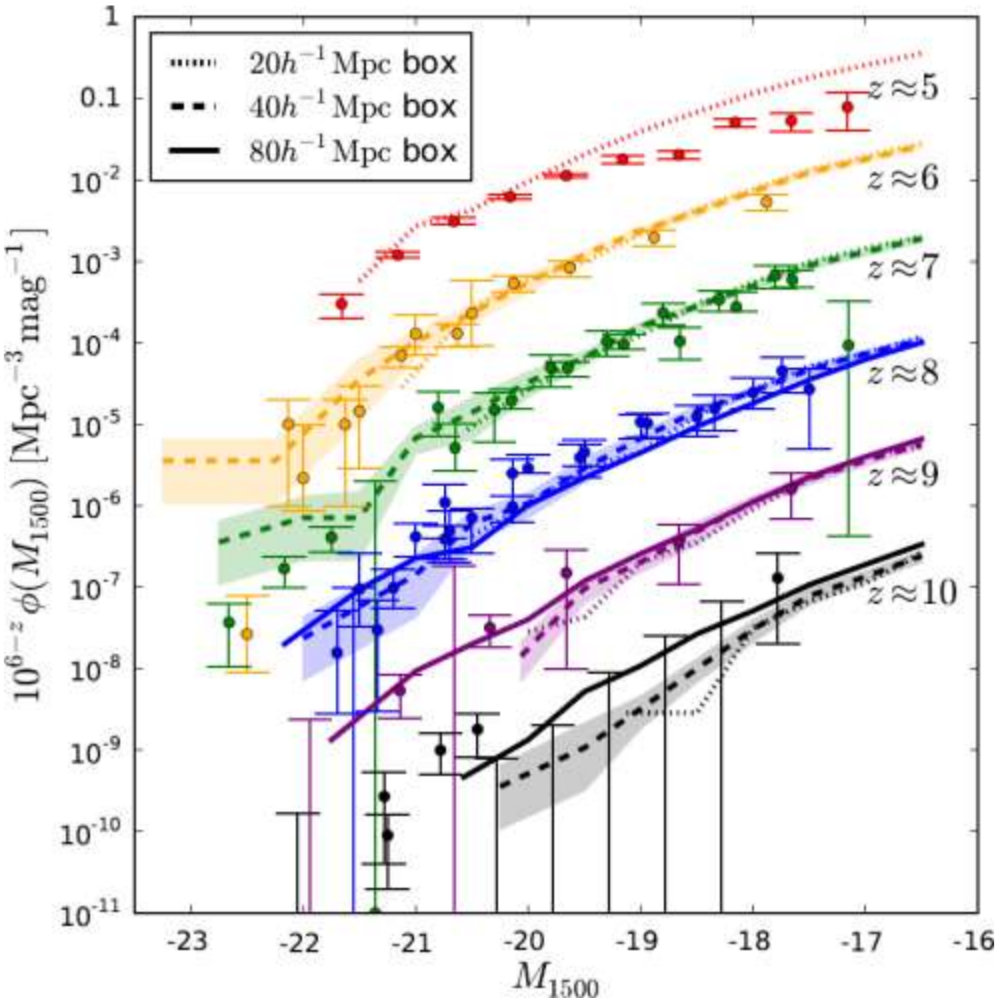
# Weak Convergence

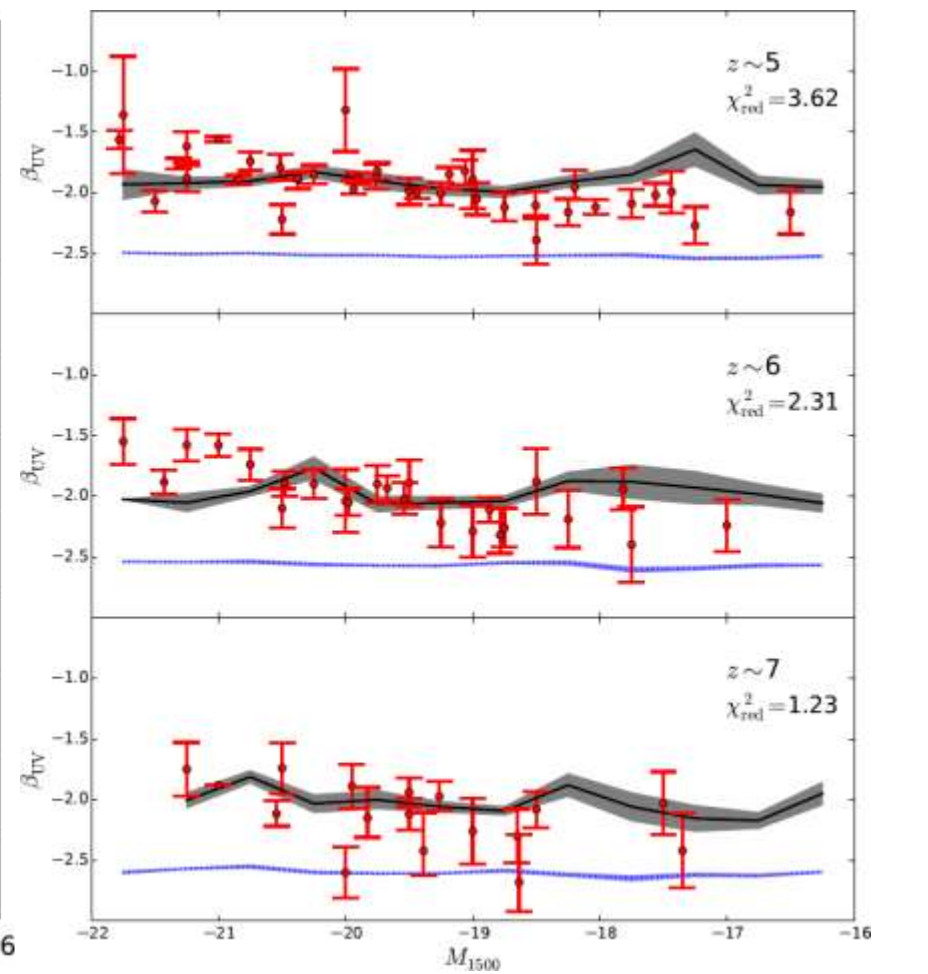
- Miracles do happen: now it works!



# It Pays To Converge

- Galaxies are ok (at all times!)



# It Pays To Converge

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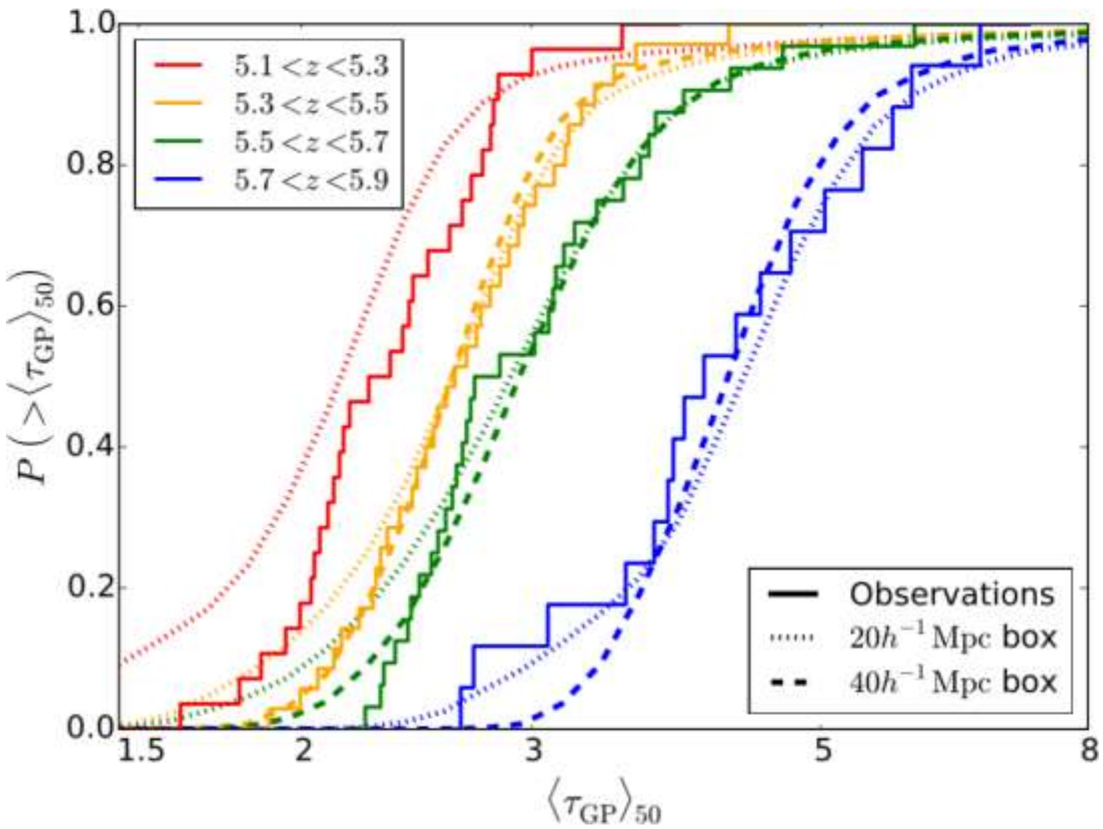
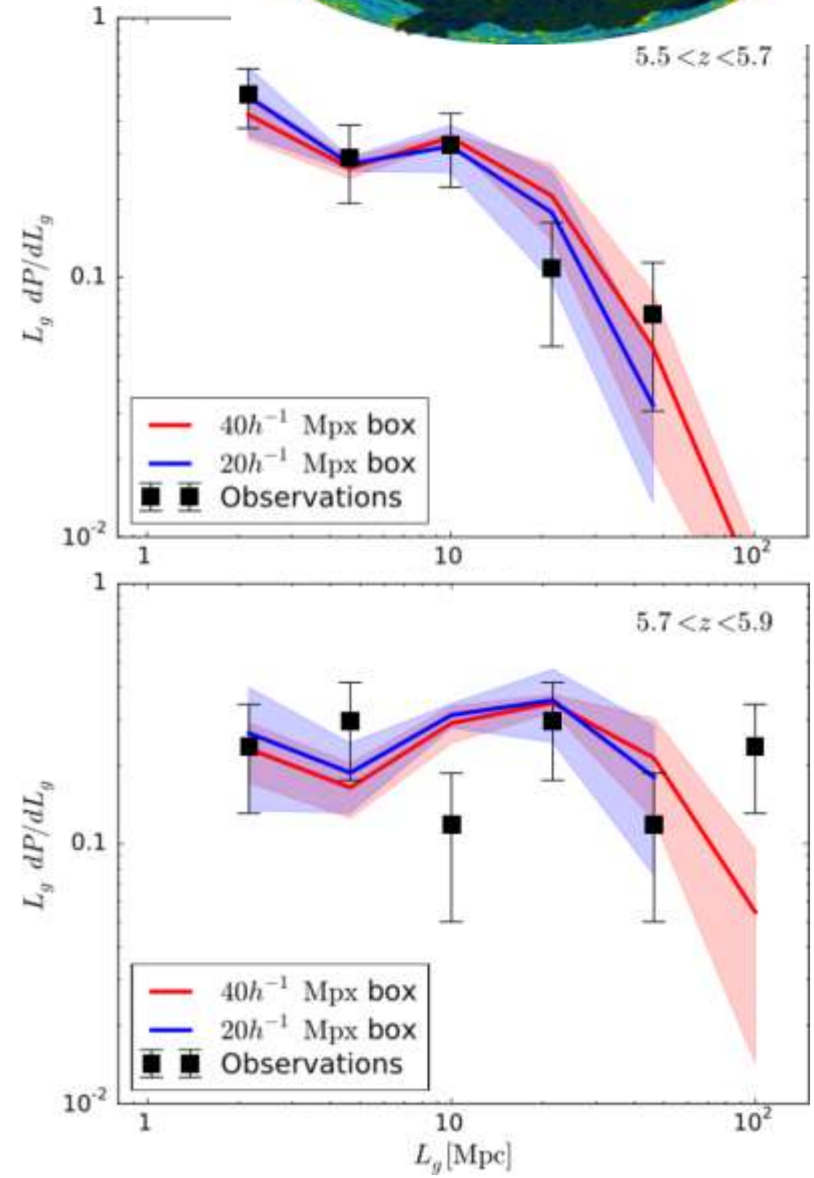
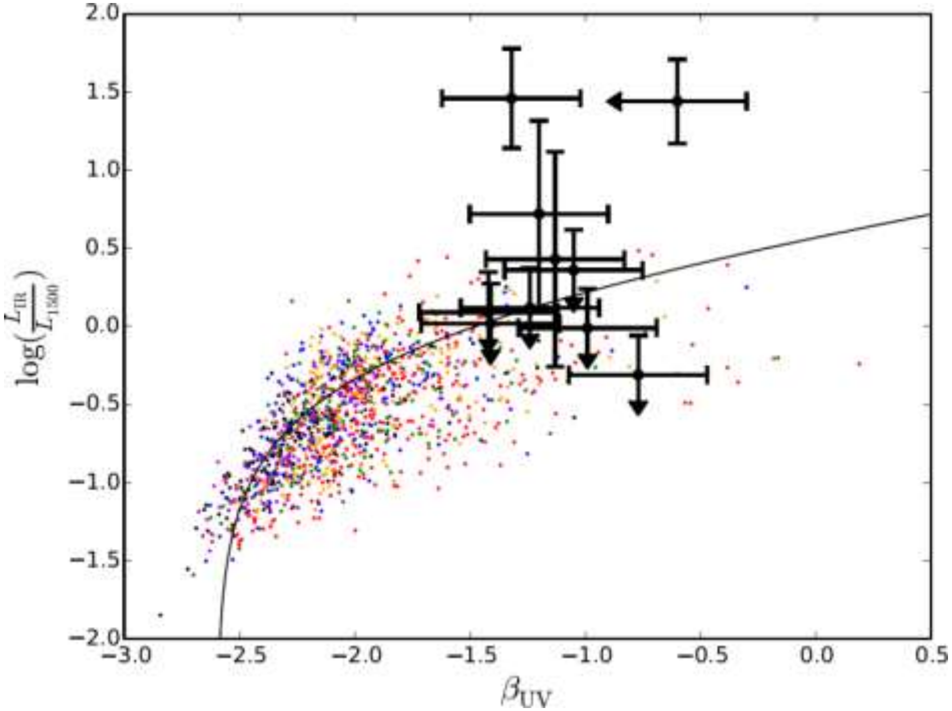
- IGM is ok.
- $\tau$ = 0.065.

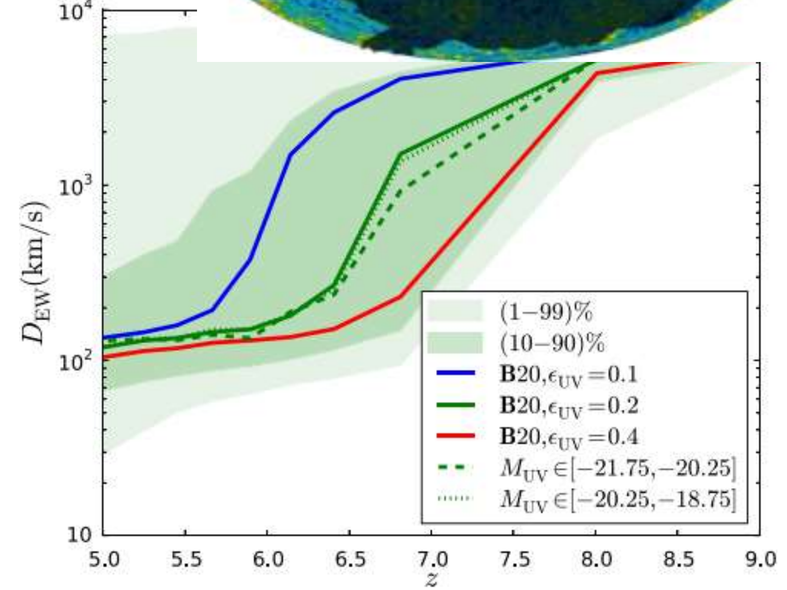


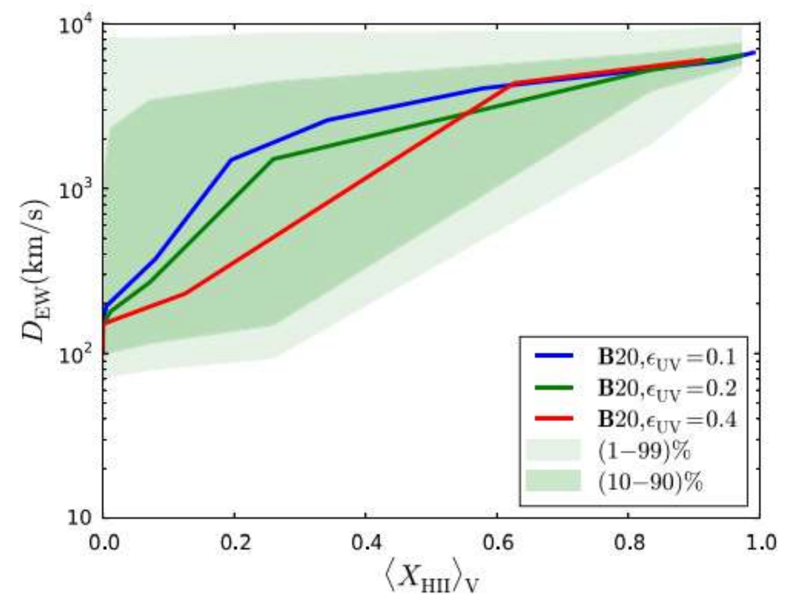



# It Pays To Converge

- LAE disappear rapidly past z~6.
- Early galaxies have enough dust.



# Conclusions

- Achieving strong convergence in realistic galaxy formation simulations is close to impossible (and, frankly, is not worth it).
- Weak convergence is a sensible approach to ensuring (some of) your results are (approximately) resolution independent.
- It may be a fool’s hope, but numerically converged solutions could be closer to reality than the ones dominated by truncation errors…